THESIS

AN APPRAISAL OF METHOD OF MOMENTS AS APPLICABLE TO WATER RESOURCE PROCESSES

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(98-PG-WRE-09)

For the Degree of

Master of Science

IN

WATER RESOURCE ENGINEERING

CENTER OF EXCELLENCE IN WATER RESOURCES ENGINEERING
UNIVERSITY OF ENGINEERING AND TECHNOLOGY
LAHORE, PAKISTAN.

2000

ABSTRACT

The method of moments is commonly used for estimating the parameters of a theoretical continuous probability distribution to fit a series of observations. It is established that parameters estimated using this definition of the method of moments are often biased and bounded. Attempts to correct for these biases have usually involved applying empirically derived correction factors to the parameter estimates.

The problem with current applications of the method of moments is basic: it does not agree with the principles of the method of moments as published in the early 1900's by Karl Pearson, especially for the range of sample sizes encountered in practice. Pearson's original definition of the method of moments is reviewed and compared to the method of moments as practised today. This comparison reveals some substantial theoretical and practical differences between the conventional method of moments and Pearson's approach.

In order to explain the discrepancies between the two methods, the history of the use of method of moments in hydrology is reviewed. This review shows that the original applications of the method in hydrology were not based directly on Pearson's work, but on a second hand account written by Pearson's colleague, William Elderton. Unfortunately, the fact that Elderton's account of the method of moments is only valid for large sample sizes appears to have been overlooked.

The literature on the method of moments in hydrology is briefly reviewed. This literature focuses on attempts to correct the problems which result when

Elderton's method is applied to the small sample sizes. All of these approaches, which range from theoretical and empirical correction factors to arbitrary weighting schemes can be viewed as attempts to treat the symptoms of the disease rather than attacking the underlying cause of the problem.

One practice which has become particularly widespread in flood frequency analysis involves fitting a probability distribution to the logarithms of a series of data rather than to the original series itself. The use of the highly non-linear logarithmic transformation is shown to result in significant biases in the estimates of the moments and quantiles of the fitted distribution in real space.

Two new approaches to parameter estimation are presented as possible solutions to the problems outlined in the thesis. The first approach is simply a continuation of traditional approaches in which some new ideas about bias correction factors are outlined. While this approach may be an improvement over current practice, it does not address the underlying problems with the conventional method of moments. The second technique is based more firmly on Pearson's definition of the method of moments. This approach calculates the moments of a sample using standard numerical integration techniques and equates these to the moments of a theoretical distribution.

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